Performance Based Learning and Assessment Task

Finding the Largest Possible Area

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

In this activity, students will determine the dimensions that will produce the largest possible area for a triangle with a set perimeter. This will lead them to the idea that a regular polygon will always produce the largest area for a given perimeter. They will then determine the shape that will produce the largest area for a given perimeter (a circle). The following mathematical concepts will be utilized:

- Perimeter/circumference and area
- Triangle Inequality Theorem
- Properties of right, isosceles, and equilateral triangles
- Pythagorean Theorem
- Properties of special right triangles (optional)

II. UNIT AUTHOR:

Abbie Brewer
Auburn Middle School/Christiansburg Middle School
Montgomery County Public Schools

III. COURSE:

Geometry

IV. CONTENT STRAND:

Triangles/Polygons and Circles

V. OBJECTIVES:

Students will be able to, given a set perimeter, find the dimensions that will produce the largest possible area for a triangle. They will be able to find this area using area formulas and the Pythagorean Theorem. Students will also be able to determine the shape that will produce the largest area for a given perimeter and find this area.

VI. REFERENCE/RESOURCE MATERIALS:

Materials include copies of the "Finding the Largest Possible Area" activity sheet, calculators, and a Smart Board or whiteboard.

VII. PRIMARY ASSESSMENT STRATEGIES:

Students will be assessed on their ability to correctly identify sets of dimensions that form a triangle with a given perimeter, calculate areas, and identify the dimensions that produce the largest area. They will also be assessed on their ability to identify dimensions of various shapes with a given perimeter, calculate areas, and identify the shape that yields the largest area. They will be required to show their calculations and provide explanations.

VIII. EVALUATION CRITERIA:

An assessment list and a benchmark of exemplary work for the activity are included in this packet.

IX. INSTRUCTIONAL TIME:

This activity is intended to take about one 90-minute block or two 45-minute periods.

Finding the Largest Possible Area

Strand
Geometry – Triangles/Polygons and Circles

Mathematical Objective(s)

Students will be able to, given a set perimeter, find the dimensions that will produce the largest possible area for a triangle. They will be able to find this area using area formulas and the Pythagorean Theorem. Students will also be able to determine the shape that will produce the largest area for a given perimeter and find this area.

Related SOL

- G.5c (determine whether a triangle exists)
- G.8 (use Pythagorean Theorem and properties of special right triangles)

NCTM Standards

- Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes
- Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Apply and adapt a variety of appropriate strategies to solve problems

Materials/Resources

- Copies of the "Finding the Largest Possible Area" activity sheet
- Calculators
- Smart Board/Whiteboard

Assumption of Prior Knowledge

- Students should know how to find the perimeter and area of a triangle, as well as other various regular polygons and circles. They should be familiar with the Triangle Inequality Theorem. They should understand properties of right, isosceles, and equilateral triangles. They should also know how to use the Pythagorean Theorem.
- Students might begin to express that there are several combinations of dimensions that will produce a given perimeter, but only a few that will actually form a triangle. They should discover while completing the task that a regular polygon will produce the largest area for a given perimeter. Students should also discover that, when given a set perimeter, the shape that will produce the largest area is a circle.

- Some students might not realize that only certain sets of three dimensions will form a triangle. They may not realize that the 3-4-5 triangle is a right triangle when finding its area. They may also be confused as to how to find the area of an isosceles or equilateral triangle. After discovering that the triangle with the largest area is an equilateral triangle, students may not make the connection that a regular polygon will always produce the largest area for a given perimeter.
- Concepts that should have been explored prior to this activity include perimeter and area of polygons and circles, the Triangle Inequality Theorem, properties of right, isosceles, and equilateral triangles, the Pythagorean Theorem, and possibly properties of special right triangles.

Introduction: Setting Up the Mathematical Task

- This task is intended to take about one 90-minute block or two 45-minute periods.
- The teacher will introduce the activity by explaining to students that they will draw upon several previously-learned concepts to solve real-world problems in geometry. These real-world problems entail finding the largest possible area of a polygon given its perimeter, and finding the shape that will produce the largest area given its perimeter. Students will be asked to brainstorm ideas. Is there a strategy that will work for all polygons?
- For the "Finding the Largest Possible Area" activity, students will be divided into groups of two or three (preferably with a mixture of low- and high-ability students). This will allow them to discuss their thinking with others, giving the low-ability students extra help and providing the high-ability students an opportunity to enhance their mathematical communication skills.
- For the activity, students will use area formulas and the Pythagorean Theorem to find the dimensions that will produce the largest possible area for a triangle given its perimeter (they should find this to be an equilateral triangle and hopefully make the connection that a regular polygon will always produce the largest area). Students will then determine the shape that will produce the largest area for a given perimeter (they should find this to be a circle).

Student Exploration

Student/Teacher Actions:

- Students will work with their group members to complete the "Finding the Largest Possible Area" activity. The teacher will rotate around the room, checking in with each group to answer/ask questions in order to facilitate learning.
- The teacher may ask the following questions to promote student thinking:
    - Why does this set of dimensions form/not form a triangle?

    - Which dimensions do you *think* will create a triangle with the largest area (before figuring it out)?
    - Is there a strategy for determining the dimensions that will produce the largest area for *any* given polygon with a set perimeter?
- The following are possible misconceptions or errors students may make and questions that can be used to address them:
    - They may not realize that only certain sets of three dimensions will form a triangle. (How can we check to see if three measurements will form a triangle?)
    - They may not realize that the 3-4-5 triangle is a right triangle when finding its area. (Think about the *type* of triangle we are dealing with. What should it look like?)
    - They may be confused as to how to find the area of an isosceles or equilateral triangle. (What pieces of information do we need to know in order to find the area of a triangle? Do we know the height? How could we find it?)
    - They may not make the connection that a regular polygon will always produce the largest area for a given perimeter. (What type of triangle produced the largest area? Now think about a quadrilateral – do you get a similar result? Can we generalize this?)

Monitoring Student Responses

- Students will communicate with their group members during the activity. They will be required to explain their thinking as they answer the questions on the activity sheet.
- The teacher will rotate amongst the groups to be sure each group is on the correct path. If a student or group is having difficulties, the teacher can ask questions to help clarify the material. If the teacher notices that multiple groups are having trouble with the same thing, he/she may bring the class together to discuss the issue. Either the teacher can provide some guidance, or he/she can call on a student who understands it to explain his/her thinking to the class. Groups that finish early may be asked to assist other groups.
- When all groups have completed the activity, the teacher will bring the class together to discuss their results. An emphasis will be placed on the different strategies students used to solve the problems. Student volunteers from each group will be given the opportunity to share their group's strategy with the class. The discussion will wrap up with student explanations of a rule to determine the dimensions that will produce the largest area for any given polygon with a set perimeter, as well as how this rule can be applied to determine the shape that will produce the largest area for a set perimeter.

Assessment List and Benchmarks

- The assessment list and a benchmark of exemplary work for the activity are included in this packet.

Finding the Largest Possible Area

1. Suppose you want to build a triangular garden, and you have 12 feet of fencing to enclose it. You want the garden to be as large as possible. Find the whole number dimensions that will produce the largest area. What is this area? Show all calculations.

2. Suppose you have 24 feet of fencing to build a garden of any shape, but you want it to be as large as possible. Explore a variety of shapes of gardens and draw your conclusions. Show all calculations and explain your reasoning.

Assessment List

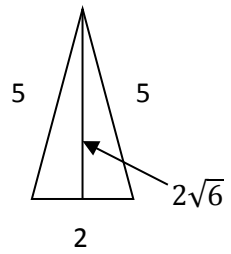
			Earned Assessment	
Number	Element	Point Value	Self	Teacher
1	The dimensions of the triangle with the largest area are identified.	2		
2	The area of the triangle is calculated.	2		
3	Calculations are provided.	2		
4	Areas of various shapes are calculated.	2		
5	Calculations are provided.	2		
6	The shape with the largest area is identified.	2		
7	Explanations are provided.	2		
8	The work is neat and presentable.	2		

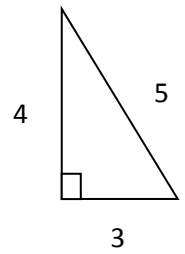
Rubric

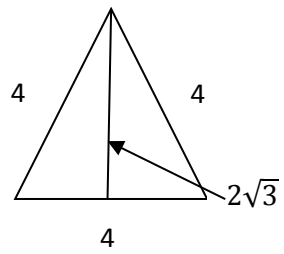
Number	Element	0	1	2
1	The dimensions of the triangle with the largest area are identified.	No dimensions or inappropriate dimensions are identified.	Dimensions are partially appropriate.	Appropriate dimensions are identified.
2	The area of the triangle is calculated.	The area is not calculated or is inappropriately calculated.	The area is partially appropriate.	The area is appropriately calculated.
3	Calculations are provided.	No calculations are provided.	Calculations are incomplete or partially appropriate.	Appropriate calculations are provided.
4	Areas of various shapes are calculated.	No areas are calculated or areas are inappropriately calculated.	Areas are insufficient or partially appropriate.	Areas of various shapes are appropriately calculated.
5	Calculations are provided.	No calculations are provided.	Calculations are incomplete or partially appropriate.	Appropriate calculations are provided.
6	The shape with the largest area is identified.	No shape or an inappropriate shape is identified.	The answer is partially appropriate.	The shape with the largest area is appropriately identified.
7	An explanation is provided.	No explanation or an inappropriate explanation is provided.	An incomplete or partially appropriate explanation is provided.	An appropriate explanation is provided.
8	The work is neat and presentable.	The work lacks neatness.	The work needs improvement.	The work is neat and presentable.

Finding the Largest Possible Area
Benchmark

1. Suppose you want to build a triangular garden, and you have 12 feet of fencing to enclose it. You want the garden to be as large as possible. Find the whole number dimensions that will produce the largest area. What is this area? Show all calculations.







$x^2 + 1^2 = 5^2$
$x^2 = 24$
$x = 2\sqrt{6}$

$A = \frac{1}{2}(2)(2\sqrt{6})$
$= 2\sqrt{6}$
≈ 4.90

$A = \frac{1}{2}(3)(4)$
$= 6$

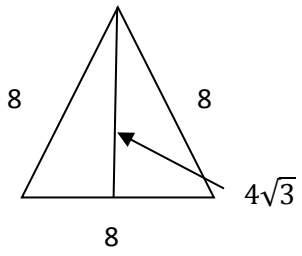
$x^2 + 2^2 = 4^2$
$x^2 = 12$
$x = 2\sqrt{3}$

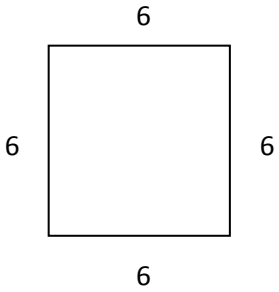
$A = \frac{1}{2}(4)(2\sqrt{3})$
$= 4\sqrt{3}$
≈ 6.93

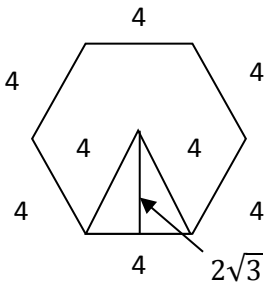
A triangle with dimensions 4 ft x 4 ft x 4 ft will have the largest area. This area is about 6.93 square feet.

2. Suppose you have 24 feet of fencing to build a garden of any shape, but you want it to be as large as possible. Explore a variety of shapes of gardens and draw your conclusions. Show all calculations and explain your reasoning.

We know from the previous problem that for any polygon, a regular one will produce the largest area. Therefore, we explore only regular shapes:







$A = \frac{1}{2}(8)(4\sqrt{3})$

$= 16\sqrt{3}$

≈ 27.71

$A = 6(6)$

$= 36$

$A = \frac{1}{2}(4)\left(2\sqrt{3}\right)(6)$

$= 24\sqrt{3}$

≈ 41.57

It appears that the area increases as we increase the number of sides. Since a circle essentially has an infinite number of "sides," we find the radius and then the area of a circle with circumference 24 feet:

$$24 = 2\pi r$$
$$\frac{12}{\pi} = r$$

$$A = \pi\left(\frac{12}{\pi}\right)^2$$
$$A \approx 45.84$$

The shape that will produce the largest garden area is a circle, with an area of approximately 45.84 square feet.